1 Quiz 3 Problem 1 (6.8.1)

Find the length L of the curve parameterized by $x(t) = 1 - t^2, y(t) = 1 + t^3$ on $0 \leq t \leq 1$.

We will use the formula $L = \int_{t=a}^{t=b} \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} dt$ so first we calculate derivatives in t:

$$\frac{dx}{dt} = x'(t) = -2t,\ \frac{dy}{dt} = y'(t) = 3t^2.$$

Now we evaluate:

$$L = \int_{t=0}^{t=1} \sqrt{(-2t)^2 + (3t^2)^2} dt = \int_0^1 \sqrt{4t^2 + 9t^4} dt \tag{1}$$

$$= \int_0^1 \sqrt{t^2(4 + 9t^2)} dt = \int_0^1 t\sqrt{4 + 9t^2} dt \tag{2}$$

$$\tag{3}$$

If $u = 4 + 9t^2$, then $du = 18t \cdot dt \implies \frac{du}{18} = t \cdot dt$, and bounds from $x = 0$ to 1 turn into bounds from $u = 4 + 9(0)^2 = 4$ to $u = 4 + 9(1)^2 = 13$.

Now,

$$L = \int_0^1 \sqrt{(2t)^2 + (3t^2)^2} dt = \int_4^{13} \sqrt{u} \cdot \frac{du}{18} du \tag{4}$$

$$= \frac{1}{18} [\frac{2}{3} u^{\frac{3}{2}}]_4^{13} \tag{5}$$

$$= \frac{2}{54} [13^{\frac{3}{2}} - 4^{\frac{3}{2}}] \tag{6}$$

$$= \frac{1}{27} [13\sqrt{13} - 4\sqrt{4}] = \frac{13\sqrt{13} - 8}{27}. \tag{7}$$

2 OR Quiz 3 Problem 2 (6.2.15)

Find the length L of the graph of $f(x)$ with $f'(x) = \sqrt{x^n - 1}$ for $2 \leq x \leq 4$, with n a positive integer.

We will use the formula $L = \int_{x=a}^{x=b} \sqrt{1 + (f'(x))^2} dx$:

$$L = \int_{x=2}^{x=4} \sqrt{1 + (\sqrt{x^n - 1})^2} dx = \int_2^4 \sqrt{1 + x^n - 1} dx \tag{8}$$

$$= \int_2^4 \sqrt{x^n} dx \tag{9}$$

$$= \int_2^4 x^{\frac{n}{2}} dx \tag{10}$$

$$= [\frac{x^{\frac{n}{2}+1}}{\frac{n}{2}+1}]_2^4 \tag{11}$$

$$= \frac{1}{\frac{n}{2}+1} [4^{\frac{n}{2}+1} - 2^{\frac{n}{2}+1}] \tag{12}$$

$$= \frac{2}{n+2} [(2^2)^{\frac{n}{2}+1} - 2^{\frac{n}{2}+1}] = \frac{2}{n+2} [2^{n+2} - 2^{\frac{n}{2}+1}] = \frac{2^{n+3} - 2^{\frac{n}{2}+2}}{n+2} \tag{13}$$

$$\tag{14}$$